A Cyclic Model of the Universe

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Why is the Universe homogeneous and isotropic on large scales? Why is it spatially flat? How did the inhomogeneities in the Universe arise that led to the formation of galaxies, and the fluctuations in the cosmic microwave background? Inflationary cosmology answers these three important questions by postulating a brief period of very rapid cosmic acceleration during the first 10^{-30} seconds after the big bang.[1–3] However, inflation does not resolve other deep questions of cosmology: What occurred at the initial singularity? How old is the Universe? How big is the Universe? And what is it's ultimate fate? What is the *raison d'etre* for the dark energy apparently present in today's Universe, revealed by recent observations[4,5] of slow cosmic acceleration? We introduce a cyclic model of the Universe that addresses all of these questions *without* introducing the ultra-high acceleration posited in inflationary models. In our model, the Universe undergoes an endless sequence of cosmic cycles each of which begins with a 'big bang' and ending with a 'big crunch'. Each big bang leads to an epoch of radiation and matter domination consistent with the standard cosmology, followed by a long period of slow cosmic acceleration (as detected in recent observations), flattening and smoothing the Universe and suppressing the density of matter, entropy and black holes. This epoch ends with contraction to the next big crunch, during which the energy density, and inhomogeneities, are developed which fuel the next cosmic cycle.

The notion of a cyclic Universe has been popular in mythology, philosophy and cosmology throughout human history.[6] In the 1930's, Richard Tolman[7] gave a discussion within the framework of general relativity assuming a closed Universe with zero cosmological constant. On top of the difficulty of having to pass through a cosmological singularity on each bounce, Tolman pointed out that entropy would undoubtedly be generated, causing the Universe to expand to a larger size in each subsequent cycle. There would be no fixed point, and the Universe would have to have originated at some finite time in the past. Consequently, Tolman's oscillating Universe models failed to represent a genuine solution to the profound philosophical problem of a 'beginning of time,' although the general notion of a bounce continued to attract interest in later decades.[8]

The cyclic model presented here is different from Tolman's in several respects. The Universe is infinite and flat, rather than finite and closed. Negative potential energy rather than spatial curvature is what causes the reversal from expansion to contraction. Before the reversal, though, the universe undergoes the usual period of radiation and matter domination, followed by a long period of accelerated expansion (presumably the acceleration that has been recently detected). The accelerated expansion naturally dilutes the density of entropy, black holes and other debris produced in the previous cycle so that the Universe is virtually empty before the next bounce. So, unlike Tolman's case, the Universe is returned to nearly pristine condition before the big crunch. During the bounce from big crunch to big bang, the density is replenished with new matter and radiation that serves as the fuel of the subsequent hot big bang phase. New quarks and leptons are created that produce new hydrogen to create new stars. After 15 billion years or so, cosmic acceleration begins anew, only to be followed by the next big crunch.

Essential Ingredients

As in inflationary cosmology, the cyclic scenario can be described in terms of the evolution of a scalar field ϕ along a potential in a four-dimensional quantum field theory. The essential differences are the form of the potential and the couplings between the scalar field and matter-radiation.

The analysis of the cyclic model follows from the action for gravity, the scalar field ϕ, and the matter and radiation fluids:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R)\right), \qquad (1)$$

where g the determinant of the metric $g_{\mu\nu}$, G is Newton's constant and $\mathcal{R}$ is the Ricci scalar. The coupling $\beta(\phi)$ between ϕ and the matter (ρ_M) and radiation (ρ_R) densities is crucial since it causes the densities to remain fnite at the big crunch/big bang transition.

The line element for a flat, homogeneous Universe is $-dt^2 + a^2 d\mathbf{x}^2$, and the equations of motion following from Eq. (1) are,

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V + \beta^4\rho_R + \beta^4\rho_M\right), \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\dot{\phi}^2 - V + \beta^4\rho_R + \frac{1}{2}\beta^4\rho_M\right), \tag{3}$$

where dot denotes t derivative and $H \equiv \dot{a}/a$. The equation of motion for ϕ is

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi} - \beta_{,\phi}\beta^3\rho_M \tag{4}$$

and the fluid equation of motion is

$$\hat{a}\frac{d\rho_i}{d\hat{a}} = a\frac{\partial\rho_i}{\partial a} + \frac{\beta}{\beta'}\frac{\partial\rho_i}{\partial\phi_i} = -3(\rho_i + p_i), \qquad i = \mathrm{M,R}, \tag{5}$$

where $\hat{a} \equiv a\beta(\phi)$ and p is the pressure of the fluid component with energy density ρ. The implicit assumption is that matter and radiation couple to $\beta^2(\phi)g_{\mu\nu}$ (with scale factor $\hat{a}$) rather than the Einstein metric $g_{\mu\nu}$ alone (or the scale factor a). Note that the radiation term in Eq. (1) is actually independent of ϕ (since $\rho_R \propto \hat{a}^{-4}$) so only ρ_M enters the ϕ equation of motion.

We assume the potential $V(\phi)$ has the following three key features (see Fig. 1). First, $V(\phi)$ must approach zero rapidly as $\phi \to -\infty$. Second, the potential must be negative for intermediate ϕ. It is when ϕ passes over this range that the density inhomogeneities are generated for the next cycle. Third, as ϕ increases, the potential must rise to a shallow plateau with a positive value V_0. An example of a potential with these properties is

$$V(\phi) = V_0(1 - e^{-c\phi})F(\phi), \tag{6}$$

where from this point onwards we adopt units in which $8\pi G = 1$. $F(\phi)$ is a function we introduce to ensure that $V(\phi) \to 0$ as $\phi \to -\infty$. (As discussed below, if ϕ is the scalar field associated with the string coupling constant, this behaviour is suggested by string theory). We take $F(\phi)$ to be nearly

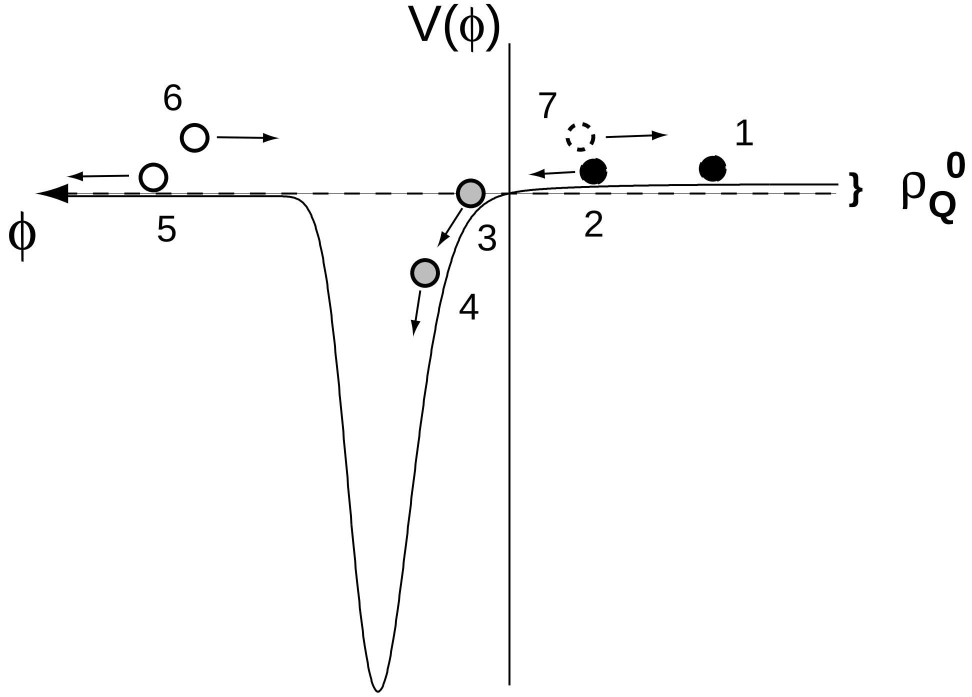

Figure 1: Schematic plot of the potential $V(\phi)$ as a function of the field ϕ. In M theory, ϕ determines the distance between branes, where $\phi \to -\infty$ as the branes collide. We define ϕ to be zero where $V(\phi)$ crosses zero and, therefore, ϕ is positive when the branes are at their maximal separation. Key features of the potential are: (a) $V(\phi) \to 0$ as $\phi \to -\infty$; (b) the negative well for $\phi < 0$; and (c) the positive plateau at $\phi > 0$. The variation from positive to negative potential energy is what causes periodic periods of expansion and contraction. The sequence of stages is described in the article.

unity for ϕ to the right of potential minimum. The detailed manner in which it tends to zero at smaller ϕ is not crucial for the main predictions of the cyclic model. A quantitative analysis of this model potential (Ref. 9) shows that a realistic cosmology can be obtained for $c \geq 10$ with V_0 chosen to be equal to today's dark energy density.

We have already mentioned that the coupling $\beta(\phi)$ is chosen so that $\hat{a}$ and, thus, the matter and radiation density are finite at $a = 0$. As explained below, this is an automatic consequence of dimensional reduction in M theory, which leads to $\beta(\phi) \sim \mathrm{e}^{-\phi\sqrt{6}}$ as $\phi \rightarrow -\infty$. The presence of $\beta(\phi)$ and the consequent coupling of ϕ to nonrelavistic matter represent a modification of Einstein's theory of general relativity. Because the scalar field ϕ evolves by a negligible amount between nucleosynthesis and today, the deviations from standard general relativity are small enough to satisfy all current cosmological constraints.[9] However, the coupling of matter to ϕ produces other potentially measurable effects including a 'fifth force' which violates the equivalence principle. Provided $(\ln\beta)_{,\phi} \ll 10^{-3}$, for today's value of ϕ, these violations are too small to be detected.[9-11] We shall assume this to be the case. Hence, the deviations from general relativity are negligible except near the big crunch/big bang transition.

The final crucial ingredient in the cyclic model is a matching rule which determines how to pass from the big crunch to the big bang. The transition occurs as $\phi \rightarrow -\infty$ and then rebounds towards positive ϕ. Motivated again by string theory (see below), we propose that some small fraction of the ϕ-field kinetic energy is converted to matter and radiation. The matching rule amounts to

$$\dot{\phi}\,\mathrm{e}^{\sqrt{3/2}\phi} \rightarrow -(1+\chi)\dot{\phi}\,\mathrm{e}^{\sqrt{3/2}\phi} \tag{7}$$

where χ is a parameter measuring the efficiency of production of radiation at the bounce. Both sides of this relation are finite at collision. In Ref. 9 it is shown that a realistic cyclic solution is obtained for small positive χ, and that this is achievable if more radiation is generated on the negative tension brane.

Stringy motivation

Superficially, the introduction of a scalar field, a potential and the couplings to matter is no more arbitrary or tuned than inflation. The cyclic model, however, has strong motivation from string theory and M-theory, and its components have a natural geometric interpretation in this context. This connection is important, giving the model quantum consistency at a deep level and tieing our scenario into the leading approach to fundamental physics. However, for low energy aspects of the model the four-dimensional field theory description is adequate, allowing the reader to follow the cyclic scenario without reference to string theory.

According to M-theory, the Universe consists of a four dimensional 'bulk' space bounded by two three-dimensional domain walls, one with positive and the other with negative tension.[13–15] The branes are free to move along the extra spatial dimension, so that they may approach and collide. (The fundamental theory is formulated in 10 spatial dimensions, but six of the dimensions are compactified on a Calabi-Yau manifold, which for our purposes can be treated as fixed, and therefore ignored). Gravity acts throughout the five dimensional space-time, but particles of our visible Universe are constrained to move along one of the branes, sometimes called the 'visible brane.' Particles on the other brane interact only through gravity with matter on the visible brane and hence behave like dark matter.

The scalar field ϕ is naturally identified with the radion field that determines the distance between branes. The potential $V(\phi)$ is the inter-brane potential caused non-perturbative virtual exchange of membranes between the boundaries. The interbrane force is what causes the branes to repeatedly collide and bounce. At large separation (corresponding to large ϕ), the force between the branes should become small, consistent with the flat plateau shown in Fig. 1. Collision corresponds to $\phi \to -\infty$. But the string coupling $g_s \propto e^{\gamma\phi}$, with $\gamma > 0$, so g_s approaches zero in this limit 16. Non-perturbative effects vanish faster than any power of g_s, for example as e^{-1/g_s^2} or e^{-1/g_s}, accounting for the prefactor $F(\phi)$ in Eq. (6).

The coupling $\beta(\phi)$ also has a natural interpretation in the brane picture. Particles reside on the branes, which are embedded in an extra dimension whose size and warp are determined by β. The effective scale factor on the branes is $\hat{a} = a\,\beta(\phi)$, not a, and $\hat{a}$ is finite at the big crunch/big bang. The function $\beta(\phi)$ is in general different for the two branes (due to the warp factor) and for different reductions of M-theory. However, the behavior $\beta(\phi) \sim \mathrm{e}^{-\phi/\sqrt{6}}$ as $\phi \to -\infty$ is universal, since at small brane separations the warp factor becomes irrelevant and one obtains the standard Kaluza-Klein result.[16,9]

Most importantly, the brane-world provides a natural resolution of the cosmic singularity.[16,9] From the brane-world perspective, the singularity is far milder than in conventional cosmology. In fact, one might say the big crunch is an illusion, since the scale factors on the branes ($\rightarrow \hat{a}$) are perfectly finite there. That is why the matter and radiation densities, and the Riemannian curvature on the branes, are finite. The only respect in which the big crunch is singular is that the one extra dimension separating the two branes momentarily disappears. Our scenario is built on the hypothesis[17] that the branes separate after collision, so the extra dimension immediately reappears. This process cannot be completely smooth, since the disappearance of the extra dimension is non-adiabatic and leads to particle production. Preliminary calculations of this effect are encouraging, since they indicate a finite density of particles is produced zero.[12] Thus the brane collision is inelastic. The matching condition, Eq. (7), parameterizes this effect. Ultimately, a well-controlled string-theoretic calculation[9,16,12] should determine the value of χ.

Hence, all the essential ingredients required for the cyclic model have a natural interpretation in the M-theory brane-world picture. As is well known, on length scales larger than the separation between branes, the higher dimensional brane-world description can be reduced to a four-dimensional field theory. Hence, except at collision itself, the cyclic scenario does not rely on stringy physics and higher dimensions. Nevertheless, the brane-world picture proves to be a useful geometrical picture for envisioning why the field behaves as it does.

Dark energy and the cyclic model

The role of dark energy in the cyclic scenario is novel. In the standard big bang and inflationary models, the recently discovered dark energy and cosmic acceleration are an unexpected surprise with no clear *raison d'etre*. In the cyclic scenario, however, not only is the source of dark energy explained, but the dark energy and its associated cosmic acceleration are actually crucial to the consistency of the model. Namely, the associated exponential expansion suppresses density perturbations and dilutes entropy, matter and black holes to negligible levels. By periodically restoring the Universe to an empty, smooth state, the acceleration causes the cyclic solution to be a stable attractor.

Right after a big bang, the scalar field ϕ is increasing rapidly. However, its motion is damped by the expansion of the Universe and ϕ essentially comes to rest in the radiation dominated phase (stage (1) in Figure 1). Thereafter it remains nearly fixed until the dark energy begins to dominate and cosmic acceleration commences. The positive potential energy density at the current value of ϕ acts as a form of quintessence,[18] a time-varying energy component with negative pressure that causes the present-day accelerated expansion. This choice entails tuning V_0, but it is the same degree of tuning required in any cosmological model (including inflation) to explain the recent observations of cosmic acceleration. In this case, because the dark energy serves several purposes, the single tuning resolves several problems at once.

The cosmic acceleration is nearly 100 orders of magnitude smaller than considered in inflationary cosmology. Nevertheless, if sustained for hundreds of e-folds (trillions of years) or more, the cosmic acceleration can flatten the Universe and dilute the entropy, black holes, and other debris created over the preceding cycle, overcoming the obstacle that has blocked previous attempts at a cyclic Universe. The number of particles in the Universe may easily be suppressed to less than one per Hubble volume before the cosmic acceleration ends. Ultimately, the scalar field begins to roll back towards $-\infty$, driving the potential to zero. The scalar field ϕ is thus the source of the currently observed acceleration, the reason why the Universe is homogeneous, isotropic and flat before the big crunch, and the root cause for the Universe reversing from expansion to contraction.

A brief tour of the cyclic universe

Putting together the various concepts that have been introduced, we can now present the sequence of events in each cycle beginning from the present epoch, stage (1) in Figure 1. The Universe has completed radiation and matter dominated epochs during which ϕ is nearly fixed. We are presently at the time when its potential energy begins to dominate, ushering in a period of slow cosmic acceleration lasting trillions of years or more, in which the matter, radiation and black holes are diluted away and a smooth, empty, flat Universe results. Very slowly the slope in the potential causes ϕ to roll in the negative direction, as indicated in stage (2). Cosmic acceleration continues until the field nears the point of zero potential energy, stage (3). The Universe is dominated by the kinetic energy of ϕ, but expansion causes this to be damped. Eventually, the total energy (kinetic plus negative potential) hits zero. From Eq. (2), the Hubble parameter is zero and the Universe is momentarily static. From Eq. (3), $\ddot{a} < 0$, so that a begins to contract. While a is nearly static, the Universe satisfies the ekpyrotic conditions for creating a scale-invariant spectrum of density perturbations.[17,19] As the field continues to roll towards $-\infty$, the scale factor a contracts and the kinetic energy of the scalar field grows. That is, gravitational energy is converted to scalar field (brane) kinetic energy during this part of the cycle. Hence, the field races past the minimum of the potential and off to $-\infty$, with kinetic energy becoming increasingly dominant as the bounce nears, stage (5). The scalar field diverges as a tends to zero. After the bounce, radiation is generated and the Universe is expanding. At first, scalar kinetic energy density ($\propto 1/a^6$) dominates over the radiation ($\propto 1/a^4$), stage (6). Soon after, however, the Universe becomes radiation dominated, stage (7). The motion of ϕ is rapidly damped away, so that it remains close to its maximal value for the rest of the the standard big bang evolution (the next 15 billion years). Then, the scalar field potential energy begins to dominate, and the field rolls towards $-\infty$, where the next big crunch occurs and the cycle begins anew.

Obtaining scale-invariant perturbations

One of the most compelling successes of inflationary theory was to obtain a nearly scale-invariant spectrum of fluctuations that can seed large-scale structure.[3] Here, the same feat is achieved using completely different physics during an ultra-slow contraction phase, (*i.e.*, stage (2) in the Figure). This alternative approach, first discussed for the ekpyrotic model, is detailed in Refs. (17) and (19).

In inflation, the density fluctuations are created by very rapid expansion, causing fluctuations on microscopic scales to be stretched to macroscopic scales.[3] In the cyclic model, the fluctuations are generated during a quasistatic, contracting Universe where gravity plays no significant role.[17] Simply because the potential $V(\phi)$ is decreasing, quantum fluctuations in ϕ are amplified as the field evolves downhill.[17,20,21] Instabilities in long-wavelength modes occur sooner than those in short wavelength modes, thereby amplifying long wavelength power and, curiously, nearly exactly mimicking the inflationary effect. In Ref. 19, it was shown how the nearly scale-invariant spectrum of fluctuations in ϕ created during the contracting phase transform into a nearly scale-invariant spectrum of density fluctuations in the expanding phase.

Current observations of large-scale structure and fluctuations of the cosmic microwave background cannot distinguish between inflation and the cyclic model because both predict a nearly scale-invariant spectrum of adiabatic, gaussian density perturbations. However, future measurements of gravitational waves may be able to do so.[17] In inflation, where gravity is paramount, quantum fluctuations in all light degrees of freedom are subject to the same gravitational effect described above. Hence, not only is there a nearly scale-invariant spectrum of energy density perturbations, but also there is a scale-invariant spectrum of gravitational waves. In the cyclic and ekpyrotic models, where the potential, rather than gravity, is the cause of the fluctuations, the only field which obtains a nearly scale-invariant spectrum is the one rolling down the potential, namely ϕ, which only produces energy density fluctuations. The direct search for gravitational waves or the search for their indirect effect on the polarization of the cosmic microwave background[22] are the crucial tests for distinguishing inflation from the cyclic model.

Cyclic solution as Cosmic Attractor

Not only do cyclic solutions exist for a range of potentials and parameters, but also they are attractors for a wide set of initial conditions. The cosmic acceleration caused by the positive potential plateau plays the critical role here. For example, suppose the scalar field is jostled and stops at a slightly different maximal value on the plateau compared to the exactly cyclic solution. The same sequence of stages ensues. The scalar field is critically damped during the exponentially expanding phase. So by the time the field reaches stage (3) where $V = 0$, it is rolling very nearly at the same rate as if it had started at $\phi = 0$, and memory of its initial position has been lost. See Ref. 9 for a detailed discussion and phase diagram. The argument suggests that it is natural to expect dark energy and cosmic acceleration following matter domination in a cyclic universe, in accordance with what has been recently observed.

The cyclic model versus inflation

Observationally, the cyclic and inflationary models are in equally good standing with current data. Conceptually, though, the cyclic model has numerous advantages.

Both inflation and the cyclic model rely on accelerated expansion. However, inflation relies on a purely hypothetical period of accelerated expansion proposed to occur at ultra-high energies, during which the universe doubles in size every 10^{-35} seconds. At present, there is no direct evidence that this acceleration occurred and the field responsible cannot be identified. The cyclic model instead relies on the acceleration that is presently observed, and which may be interpreted as evidence for the ϕ field. In this sense, the cyclic model has one less *ad hoc* assumption.

In the inflationary picture, most of the volume of the Universe is completely unlike what we see. Even when inflation ends in one region, such as our own, it continues in others. Because of the superluminal expansion rate of the remaining inflating regions, they occupy most of the physical volume of the Universe. Inflationary remnants, such as our region of the Universe, represent an infinitessimal fraction. By contrast, the cyclic model is one in which the local Universe is typical of the Universe as a whole. All or almost all regions of the Universe are undergoing the same sequence of cosmic events and most of the time is spent in the radiation, matter, and dark energy dominated phases. In this sense, the cyclic model is more economical than inflationary cosmology.

In the production of perturbations, the inflationary mechanism relies on stretching modes whose wavelength is initially exponentially sub-Planckian, to macroscopic scales. Quantum gravity effects in the initial state are highly uncertain, and inflationary predictions may therefore be highly sensitive to sub-Planckian physics. In contrast, perturbations in the cyclic model are generated when the modes have wavelengths of thousands of kilometers, using macroscopic physics insensitive to quantum gravity effects.

The cyclic model deals directly with the cosmic singularity, explaining it as a transition from a contracting to an expanding phase. As discussed, string theory and M-theory provide evidence for this point-of-view. Although inflation does not address the cosmic singularity problem directly, it does rely implicitly on the opposite assumption: that the big bang is the beginning of time and that the Universe emerges in a rapidly expanding state. Inflating regions with high potential energy expand more rapidly and dominate the Universe. If there is a pre-existing contracting phase, then the high potential energy regions collapse and disappear before the expansion phase begins. Hence, progress on the singularity problem may ultimately decide between the two cosmologies.

Finally, the cyclic model is a complete model of cosmic history, whereas inflation is only a theory of cosmic history following an assumed initial creation event. Hence, the cyclic model has, in principle, far more explanatory and predictive power. For example, we have already emphasized how the cyclic model leads naturally to the prediction of quintessence and cosmic acceleration, explaining them as essential elements of an eternally repeating Universe.

The cyclic model provides a fascinating new outlook on the cosmological constant problem. Historically, the problem is often assumed to mean that one must explain why the vacuum energy of the ground state is zero. In the cyclic model, the vacuum energy of the ground state is not zero. It is negative and its magnitude is large, as is obvious from Figure 2. If the Universe begins in the ground state, the negative cosmological constant will cause rapid recollapse, as expected for an anti-deSitter phase. In the cyclic scenario, though, we have shown that the Universe hovers above the ground state from cycle to cycle, bouncing from one side of the potential well to the other but spending most time on the positive energy side. There remains the important challenge of explaining why the the current potential energy is so small. The value depends on both the shape of the potential curve and the precise transfer of energy and momentum at the bounce.[9] Perhaps explaining the value will be an issue as knotty as the cosmological constant problem, or perhaps the conditions will prove easier to satisfy. What is certain, though, is that the problem is shifted from conventional tuning of vacuum energy. More generally, the notion of hovering around the true vacuum state means that our present condition, such as the degree of supersymmetry breaking, is fixed cosmologically rather than by minimizing the potential, which has implications for many aspects in the design of phenomenological supersymmetric particle physics models.

Reviewing the overall scenario and its implications, what is most remarkable is that the cyclic model can differ so much from the standard picture in terms of the origin of space and time and the sequence of cosmic events that lead to our current Universe; and, yet, the model requires no more assumptions or tunings (and by some measures less), has more explanatory power, and matches current observations to the same exquisite degree of precision.

Of course, the ultimate arbiter will be Nature. Specifically, measurements of the stochastic gravitational background is the decisive way to distinguish the two scenarios. Another generic prediction of the cyclic model regards the ratio of the pressure to the energy density of the dark energy that is causing the current cosmic acceleration. In the cyclic picture, the dark energy is due to the scalar field ϕ which has been fixed during the radiation and matter era, but is beginning to roll downhill as the Universe becomes dark energy dominated and the expansion begins to accelerate. For a static field, the ratio of pressure to energy density is -1, but this ratio increases as the field begins to roll. Hence, measuring the ratio today and perhaps its time-variation are further consistency checks of the cyclic picture. In the interim, it appears that we now have two disparate possibilities: a Universe with a definite beginning and a Universe that is made and remade forever.

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