Gravitational Condensate Stars: An Alternative to Black Holes

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A new solution for the endpoint of gravitational collapse is proposed. By extending the concept of Bose-Einstein condensation to gravitational systems, a cold, compact object with an interior de Sitter condensate phase and an exterior Schwarzschild geometry of arbitrary total mass M is constructed. These are separated by a phase boundary with a small but finite thickness ℓ of fluid with eq. of state $p = +\rho c^2$, replacing both the Schwarzschild and de Sitter classical horizons. The new solution has no singularities, no event horizons, and a global time. Its entropy is maximized under small fluctuations and is given by the standard hydrodynamic entropy of the thin shell, which is of order $k_{_B}\ell Mc/\hbar$, instead of the Bekenstein-Hawking entropy, $S_{_{BH}} = 4\pi k_{_B} GM^2/\hbar c$. Unlike black holes, a collapsed star of this kind is thermodynamically stable and has no information paradox.

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Introduction. The vacuum Einstein eqs. of classical general relativity possess a well-known solution for an isolated mass M, with the static, spherically symmetric line element,

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{h(r)} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) , \qquad (1)$$

where the functions $f(r)$ and $h(r)$ are given by

$$f(r) = h(r) = 1 - \frac{2GM}{r} , \qquad (2)$$

in units where $c = 1$. The dynamical singularity of this Schwarzschild metric at $r = 0$ with its infinite tidal forces clearly signals a breakdown of the vacuum Einstein eqs. The kinematical singularity at the Schwarzschild radius $R_{_S} = 2GM$ is of a different sort, corresponding to an infinite blue shift of the frequency of an infalling light wave with respect to its frequency far from the black hole. Since the curvature tensor is finite at $r = R_{_S}$, the singularity of the metric (1)-(2) there can be removed by a suitable change of coordinates in the classical theory. A classical point test particle freely falling through the event horizon experiences nothing special at $r = R_{_S}$.

The physics at the event horizon may be quite different when $\hbar \neq 0$. Consider the same photon of finite asymptotic frequency ω far from the black hole. Even if $\hbar\omega$ is arbitrarily small, the infrared photon acquires a local energy $E = \hbar\omega f^{-\frac{1}{2}}$, which diverges at $r = R_{_S}$. Hence there is no *a priori* small parameter controlling deformation of the local geometry near $r = R_{_S}$ due to quantum effects.

In the semi-classical approximation, when a massless field such as that of the photon is quantized in the fixed Schwarzschild background, one finds that the black hole radiates these quanta with a thermal spectrum at the asymptotic Hawking temperature, $T_{_H} = \hbar/8\pi k_B GM$ [1]. It is usually assumed that the backreaction effect of this radiation on the classical geometry must be quite small. However, detailed calculations of the energy-momentum of the radiation show that its $\langle T^t_{\ t}\rangle$ and $\langle T^r_{\ r}\rangle$ components have an f^{-1} infinite blue shift factor at the horizon [2]. The wavelengths contributing to these quantum stresses are of order $R_{_S}$ and hence are non-local on the scale of the hole. One has only to insert such a diverging energy density and pressure into the semi-classical Einstein eqs. (3) below to see that the geometry near $r = R_{_S}$ is changed significantly from the classical Schwarzschild form. Unlike the classical kinematic singularity in (2), such non-local semi-classical backreaction effects near $R_{_S}$ cannot be removed by a local coordinate transformation.

Furthermore, the inverse dependence of $T_{_H}$ on M implies that a black hole in thermal equilibrium with its own Hawking radiation has negative specific heat and therefore is unstable to thermodynamic fluctuations [3]. Energy conservation plus a thermal radiation spectrum also imply that the black hole has an enormous entropy, $S_{_{BH}} \simeq 10^{77} k_{_B} (M/M_\odot)^2$ [4], far in excess of a typical stellar progenitor. The associated information paradox has been conjectured to require an alteration in the principles of relativity, or quantum mechanics, or both.

In lieu of such fundamental revision of the laws of physics, it is reasonable to examine alternatives to the strictly classical view of the event horizon as a harmless kinematic singularity, when $\hbar \neq 0$ and the quantum wavelike properties of matter are taken into account. In earlier investigations which attempted to include the backreaction of the Hawking radiation in a self-consistent way, the entropy arises entirely from the radiation fluid [5,6]. In fact, $S = 4\,\frac{\kappa+1}{7\kappa+1} S_{_{BH}}$, for a fluid with the eq. of state, $p = \kappa\rho$, becoming equal to the Bekenstein-Hawking entropy for $\kappa = 1$. Despite such intriguing features, these models have huge (Planckian) energy densities near $R_{_S}$ and a negative mass singularity at $r = 0$, so that the Einstein eqs. are not reliable in the interior region.

On the other hand, instead of abandoning quantum mechanics, demanding that its principles apply to selfgravitating systems implies that their specific heat $\propto \langle(\Delta E)^2\rangle$ must be positive. This leads to the proposal that quantum black hole entropy is carried *not* by Planckian but by *soft* quanta with $E \sim \hbar/GM$ [7]. More recently another proposal for incorporating quantum effects has been made, *viz.* that the horizon becomes a critical surface of a phase transition in the quantum theory, supported by an interior region with eq. of state, $p = -\rho < 0$ [8]. Such a vacuum eq. of state, first proposed by Gliner [9] for the endpoint of gravitational collapse, is equivalent to a positive cosmological constant in Einstein's eqs., and does not satisfy the energy condition $\rho + 3p \geq 0$ needed to prove the classical singularity theorems.

In this Letter we show that an explicit static solution of Einstein's eqs. taking quantum considerations into account exists, with the critical surface of ref. [8] replaced by a thin shell of ultra-relativistic fluid of soft quanta obeying $\rho = p$. Such a solution, lacking a singularity and an horizon is significant because it provides a stable alternative to black holes as the endpoint of gravitational collapse, possibly with different observational signatures.

The assumption required for this solution to exist is that gravity undergoes a vacuum rearrangement phase transition in the vicinity of $r = R_{_S}$. Specifically, in this region quantum fluctuations on the scale $R_{_S}$ dominate the $T^t_{\ t} \sim T^r_{\ r}$ components of the stress tensor, which grow so large that the eq. of state approaches the most extreme one allowed by causality, $\rho = p$. As this causal limit is reached, the interior spacetime becomes unstable to the formation of a new kind of gravitational Bose-Einstein condensate (GBEC) described by a non-zero macroscopic order parameter Ψ. If $|\Psi|^2$ is a constant scalar, it must couple to Einstein's eqs. in the same way as a cosmological term, and the eq. of state of the interior region must be $\rho_{_V} = -p_{_V} = V(|\Psi|^2)$. A suggestion for the effective theory incorporating the effects of quantum anomalies that could give rise to both this interior GBEC phase and the $\rho = p$ shell has been presented elsewhere [10]. Here we forego any discussion of the details of the phase transition or collapse process and consider only the compact, stable endpoint of gravitational collapse, by solving the static Einstein's eqs. with the specified eqs. of state. For a recent review of investigations of other non-singular quasi-black-hole (QBH) models see ref. [11].

Solution of Eqs. The eqs. to be solved are the Einstein eqs. for a perfect fluid at rest in the coordinates (1), *viz.*

$$-G^t_{\ t} = \frac{1}{r^2}\frac{d}{dr}\left[r\,(1-h)\right] = -8\pi G\, T^t_{\ t} = 8\pi G\rho\,, \tag{3a}$$

$$G^r_{\ r} = \frac{h}{rf}\frac{df}{dr} + \frac{1}{r^2}\,(h-1) = 8\pi G\, T^r_{\ r} = 8\pi G p\,, \tag{3b}$$

together with the conservation eq.,

$$\nabla_a T^a_{\ r} = \frac{dp}{dr} + \frac{\rho + p}{2f}\frac{df}{dr} = 0\,, \tag{4}$$

which ensures that the other components of the Einstein eqs. are satisfied. These three first order eqs. for the four unknown functions, f, h, ρ, and p become closed when an eq. of state for the fluid, relating p and ρ is specified. Because of the considerations above we allow for three different regions with the three different eqs. of state,

$$\begin{array}{lllll} \text{I.} & \text{Interior}: & 0 \leq r < r_1\,, & \rho = -p\,, \\ \text{II.} & \text{Shell}: & r_1 < r < r_2\,, & \rho = +p\,, \\ \text{III.} & \text{Exterior}: & r_2 < r\,, & \rho = p = 0\,. \end{array} \tag{5}$$

At the interfaces $r = r_1$ and $r = r_2$, we require the metric coefficients r, f and h to be continuous, although the first derivatives of f, h and p must be discontinuous from the first order eqs. (3) and (4).

In the interior region $\rho = -p$ is a constant from (4). Let us call this constant $\rho_{_V} = 3H_0^2/8\pi G$. If we require that the origin is free of any mass singularity then the interior is determined to be a region of de Sitter spacetime in static coordinates, *i.e.*

$$\text{I.} \qquad f(r) = C\,h(r) = C\,(1 - H_0^2\, r^2)\,, \quad 0 \leq r \leq r_1\,. \tag{6}$$

The unique solution in the exterior vacuum region which approaches flat spacetime as $r \to \infty$ is a region of Schwarzschild spacetime (2), *viz.*

$$\text{III.} \qquad f(r) = h(r) = 1 - \frac{2GM}{r}\,, \qquad r_2 \leq r\,. \tag{7}$$

The integration constants C, H_0 and M are arbitrary.

The only non-vacuum region is region II. Let us define the dimensionless variable w by $w \equiv 8\pi G r^2 p$, so that eqs. (3)-(4) with $\rho = p$ may be recast in the form,

$$\frac{dr}{r} = \frac{dh}{1 - w - h}\,, \tag{8a}$$

$$\frac{dh}{h} = -\frac{1 - w - h}{1 + w - 3h}\,\frac{dw}{w}\,. \tag{8b}$$

together with $pf \propto wf/r^2$ a constant. The first of eqs. (8) is equivalent to the definition of the (rescaled) Tolman mass function by $h = 1 - \mu/r$ and $d\mu(r) = 2\,G\,dm(r) = 8\pi G\,\rho r^2\,dr = w\,dr$ within the shell. The second eq. (8b) can be solved only numerically in general. However, it is possible to obtain an analytic solution in the thin shell limit, $0 < h \ll 1$, for in this limit we can set h to zero on the right side of (8b) to leading order, and integrate it immediately to obtain

$$h \equiv 1 - \frac{\mu}{r} \simeq \epsilon\,\frac{(1+w)^2}{w} \ll 1\,, \tag{9}$$

in region II, where ϵ is an integration constant. Because of the condition $h \ll 1$ we require $\epsilon \ll 1$, with w of order unity. Making use of eqs. (8) and (9) we have

$$\frac{dr}{r} \simeq -\epsilon\, dw\,\frac{(1+w)}{w^2}\,. \tag{10}$$

Because of the approximation $\epsilon \ll 1$, the radius r hardly changes within region II, and dr is of order $\epsilon\, dw$. The final unknown function f is given by $f = (r/r_1)^2(w_1/w)f(r_1) \simeq (w_1/w)f(r_1)$ for small ϵ.

Now requiring continuity of the metric coefficients f and h at both r_1 and r_2 gives the conditions,

$$h(r_1) = 1 - H_0^2 r_1^2 \simeq \epsilon\, \frac{(1+w_1)^2}{w_1}\,, \tag{11a}$$

$$h(r_2) = 1 - \frac{2GM}{r_2} \simeq \epsilon\, \frac{(1+w_2)^2}{w_2}\,, \tag{11b}$$

$$\frac{f(r_2)}{h(r_2)} = 1 \simeq \frac{w_1}{w_2}\frac{f(r_1)}{h(r_2)} = C\left(\frac{1+w_1}{1+w_2}\right)^2 \tag{11c}$$

Together with r_2/r_1 from the solution of (10) this gives four relations among the eight integration constants $(r_1, r_2, w_1, w_2, H_0, M, C, \epsilon)$. Hence the first four can be eliminated in favor of H_0, M, C and $\epsilon \ll 1$, and we have a four parameter family of static solutions. Assuming that (r_1, r_2, w_1, w_2) remain finite as $\epsilon \to 0$, *i.e.* are of order ϵ^0, we obtain from (10) and (11) that $r_1 \simeq H_0^{-1} \simeq r_2 \simeq 2GM$, but $1 - H_0\, r_1,\ r_2 - 2GM, w_1 - w_2$ and $C - 1$ are all of order ϵ, while $r_2 - r_1$ is $\mathcal{O}(\epsilon^2)$.

Principal Features. If $\epsilon > 0$ both f and h are of order ϵ and approximately constant in region II, but are nowhere vanishing. Hence there is no event horizon, and t is a global time. A photon experiences a very large, $\mathcal{O}(\epsilon^{-\frac{1}{2}})$ but finite blue shift in falling into the shell from infinity. The proper thickness of the shell,

$$\ell = \int_{r_1}^{r_2} dr\, h^{-\frac{1}{2}} \simeq R_S\epsilon^{\frac{1}{2}} \int_{w_2}^{w_1} dw\, w^{-\frac{3}{2}} \sim \epsilon^{\frac{3}{2}} R_S \tag{12}$$

is small compared to R_S. Although ℓ is arbitrary here, presumably it is fixed by microscopic physics and is not greater than the Planck length by more than a few orders of magnitude. If we assume that ℓ is a fixed multiple of the Planck length and independent of M, then $\epsilon \sim (\ell/GM)^{\frac{2}{3}} \sim (M_{Pl}/M)^{\frac{2}{3}} \simeq 10^{-25\pm1}$ for a solar mass object, which certainly justifies the small ϵ approximation in this case. With ℓ fixed, $H_0^{-1} \simeq 2GM$ and $C = 1 + \mathcal{O}(\epsilon)$, the only remaining free parameter is M.

The entropy of the shell is obtained from the eq. of state, $p = \rho = (a^2/8\pi G)(k_B T/\hbar)^2$, where we have introduced G for dimensional reasons so that a^2 is a dimensionless constant. By the standard thermodynamic relation, $Ts = p + \rho$ for a relativistic fluid with zero chemical potential, and the local specific entropy density $s(r) = a^2 k_B^2 T(r)/4\pi\hbar^2 G = a(k_B/\hbar)(p/2\pi G)^{\frac{1}{2}}$ for local temperature $T(r)$. Thus $s = (a k_B/4\pi\hbar G r)\, w^{\frac{1}{2}}$ and the entropy of the fluid within the shell is

$$S = 4\pi \int_{r_1}^{r_2} s\, r^2\, dr\, h^{-\frac{1}{2}} \simeq \frac{a k_B R_S^2}{\hbar G}\ \epsilon^{\frac{1}{2}}\ \ln\left(\frac{w_1}{w_2}\right)\,. \tag{13}$$

Using $w_1/w_2 = 1 + \mathcal{O}(\epsilon)$ and (12) this is of order

$$S \sim a\, k_B\, \frac{M}{\hbar}\, R_S \epsilon^{\frac{3}{2}} \sim a\, k_B\, \frac{M\ell}{\hbar} \ll S_{BH}\,. \tag{14}$$

Since the interior region I has $\rho_V = -p_V$, Ts vanishes there, as could be anticipated for a GBEC described by a macroscopic single quantum state. The entropy of the entire quasi-black hole (QBH) is given then by (13) or (14), which is of order $10^{38} k_B(\ell/L_{Pl})$ for a solar mass object. This is some 38 orders of magnitude smaller than the Bekenstein-Hawking entropy for the same mass M, and 20 ± 1 orders of magnitude lower than a typical stellar progenitor, which have entropies in the range of $10^{57} k_B$ to $10^{59} k_B$ for $M/m_N \sim 10^{57}$ nucleons. Since w is of order unity in the shell while $r \simeq R_S$, the *local* temperature of the fluid within the shell is of order $T_H \sim \hbar/k_B GM$, *i.e.* the typical quanta are *soft*. Because of the absence of an event horizon, the QBH does not emit Hawking radiation. In fact, the thermal wavelength of the soft quanta is of order the linear dimension R_S of the object, so that quantum zero point fluctuations and finite size effects are competitive with classical thermodynamics. If w is somewhat smaller than unity, the shell need not emit any thermal radiation at all. A discussion of the corrections to the classical hydrodynamic relations this entails must await a fuller quantum treatment. With no thermal emission the GBEC remnant then would be both ultracold and completely dark.

The extremely cold radiation fluid in the shell is confined to region II by the surface tensions at the timelike interfaces r_1 and r_2. These arise from the pressure discontinuities, $\Delta p_1 \simeq H_0^2\,(3 + w_1)/8\pi G$ and $\Delta p_2 \simeq -w_2/32\pi G^3 M^2$, and are calculable by the Israel junction conditions [12,13]. We find the non-zero angular components of the surface tension to leading order in ϵ,

$$\sigma^\theta{}_\theta = \sigma^\phi{}_\phi \simeq \frac{1}{32\pi G^2 M}\frac{(3+w_1)}{(1+w_1)}\left(\frac{w_1}{\epsilon}\right)^{\frac{1}{2}}\,, \tag{15a}$$

$$\sigma^\theta{}_\theta = \sigma^\phi{}_\phi \simeq -\frac{1}{32\pi G^2 M}\frac{w_2}{(1+w_2)}\left(\frac{w_2}{\epsilon}\right)^{\frac{1}{2}}\,. \tag{15b}$$

at r_1 and r_2 respectively. The signs correspond to the inner surface at r_1 exerting an outward force and the outer surface at r_2 exerting an inward force, *i.e.* both surface tensions exert a confining pressure on the shell region II. The time component $\sigma^t{}_t = 0$, corresponding to vanishing contribution to the Tolman mass function $m(r)$ at the two interfaces. Since $\epsilon^{-\frac{1}{2}} \sim (M/M_{pl})^{\frac{1}{3}}$, the surface tensions (15) are of order $M^{-\frac{2}{3}}$ and far from Planckian. Hence the matching of the metric at the phase interfaces r_1 and r_2, analogous to that across stationary shocks in hydrodynamics, should be reliable. Resolving the interfaces will require going beyond Einstein's eqs. to a more microscopic description of the quantum phase transition.

The energy within the shell (as measured at infinity),

$$E_{\rm II} = 4\pi \int_{r_1}^{r_2} \rho\, r^2 dr \simeq \epsilon M \int_{w_2}^{w_1} \frac{dw}{w}\,(1+w) \sim \epsilon^2 M \tag{16}$$

is also extremely small. Hence essentially all the mass of the object comes from the energy density of the vacuum condensate in the interior, even though the shell is responsible for all of its entropy.

Stability. In order to be a physically realizable endpoint of gravitational collapse, any quasi-black hole candidate must be stable [7]. Since only the region II is non-vacuum, with a 'normal' fluid and a positive heat capacity, it is clear that the solution is thermodynamically stable. The most direct way to demonstrate this stability is to work in the microcanonical ensemble with fixed total M, and show that the entropy functional,

$$S = \frac{ak_{_B}}{\hbar G}\int_{r_1}^{r_2} r\,dr\,\left(\frac{d\mu}{dr}\right)^{\frac{1}{2}}\left(1-\frac{\mu(r)}{r}\right)^{-\frac{1}{2}}\,, \tag{17}$$

is maximized under all variations of $\mu(r)$ in region II with the endpoints (r_1, r_2), or equivalently (w_1, w_2) fixed.

The first variation of this functional with the endpoints r_1 and r_2 fixed vanishes, *i.e.* $\delta S = 0$ by the Einstein eqs. (3) for a static, spherically symmetric star. Thus any solution of eqs. (3)-(4) is guaranteed to be an extremum of S [14]. This is also consistent with regarding Einstein's eqs. as a form of hydrodynamics, strictly valid only for the long wavelength, gapless excitations in gravity.

The second variation of (17) is

$$\delta^2 S = \frac{ak_{_B}}{4\hbar G}\int_{r_1}^{r_2} r\,dr\,\left(\frac{d\mu}{dr}\right)^{-\frac{3}{2}} h^{-\frac{1}{2}}\,\times \left\{-\left[\frac{d(\delta\mu)}{dr}\right]^2 + \frac{(\delta\mu)^2}{r^2h^2}\frac{d\mu}{dr}\left(1+\frac{d\mu}{dr}\right)\right\}\,, \tag{18}$$

when evaluated on the solution. Associated with this quadratic form in $\delta\mu$ is a second order linear differential operator $\mathcal{L}$ of the Sturm-Liouville type, *viz.*

$$\frac{d}{dr}\left\{r\left(\frac{d\mu}{dr}\right)^{-\frac{3}{2}} h^{-\frac{1}{2}}\frac{d\chi}{dr}\right\} + \frac{h^{-\frac{5}{2}}}{r}\left(\frac{d\mu}{dr}\right)^{\frac{1}{2}}\left(1+\frac{d\mu}{dr}\right)\chi \equiv \mathcal{L}\chi\,. \tag{19}$$

This operator possesses two solutions satisfying $\mathcal{L}\chi_{_0} = 0$, obtained by variation of the classical solution, $\mu(r; r_1, r_2)$ with respect to the parameters (r_1, r_2). Since these correspond to varying the positions of the interfaces, $\chi_{_0}$ does not vanish at (r_1, r_2) and neither function is a true zero mode. For example, it is easily verified that one solution is $\chi_{_0} = 1 - w$. However, we may set $\delta\mu \equiv \chi_{_0}\,\psi$, where ψ does vanish at the endpoints and insert this into the second variation (18). Integrating by parts, using the vanishing of $\delta\mu$ at the endpoints and $\mathcal{L}\chi_{_0} = 0$ gives

$$\delta^2 S = -\frac{ak_{_B}}{4\hbar G}\int_{r_1}^{r_2} r\,dr\,\left(\frac{d\mu}{dr}\right)^{-\frac{3}{2}} h^{-\frac{1}{2}}\chi_{_0}^2\left(\frac{d\psi}{dr}\right)^2 < 0\,. \tag{20}$$

Thus the entropy of the solution is maximized with respect to radial variations that vanish at the endpoints, *i.e.* those with fixed total energy. Since deformations with non-zero angular momentum decrease the entropy even further, stability under radial variations is sufficient to demonstrate that the solution is stable to all small perturbations. In the context of a hydrodynamic treatment, thermodynamic stability is also a necessary and sufficient condition for the *dynamical* stability of a static, spherically symmetric solution of Einstein's eqs. [14].

Conclusions. A compact, non-singular solution of Einstein's eqs. has been presented here as a possible stable alternative to black holes for the endpoint of gravitational collapse. Realizing this alternative requires that a quantum gravitational vacuum phase transition intervene before the classical event horizon can form. Although only the static spherically symmetric case has been considered, it is clear on physical grounds that axisymmetric rotating solutions should exist as well. Since the entropy of these objects is some 20 orders of magnitude smaller than that of a typical stellar progenitor, there is no entropy paradox and instead a violent process of entropy shedding, as in a supernova, is needed to produce a cold GBEC or 'grava(c)star' remnant. The shell with its maximally stiff eq. of state $p = \rho c^2$, where the speed of light is equal to the speed of sound could be expected to produce outgoing shock fronts when struck. These may serve to distinguish gravastars from black holes observationally, and possibly provide a more efficient central engine for energetic astrophysical sources. The spectra of gravitational radiation from a struck gravastar should bear the imprint of its fundamental frequencies of vibration. Finally, we note that the interior de Sitter region with $p = -\rho c^2$ may be interpreted also as a cosmological spacetime, with the horizon of the expanding universe replaced by a quantum phase interface.

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